



DOI: <https://doi.org/10.15688/mpcm.jvolsu.2022.2.4>

UDC 535.3, 538.9

LBC 22.343.4



Submitted: 21.03.2022

Accepted: 22.04.2022

**EXTREMELY SHORT OPTICAL PULSES IN CARBON  
NANOTUBES TAKING INTO ACCOUNT THE MULTIPHOTON  
ABSORPTION IN THE PRESENCE OF RANDOM STRESS<sup>1</sup>**

**Sergey V. Belibikhin**

Postgraduate Student, Department of Forensic Science and Physical Materials Science,  
Volgograd State University  
belibihin@gmail.com  
Prosp. Universitetsky, 100, 400062 Volgograd, Russian Federation

**Natalia N. Konobeeva**

Doctor of Physical and Mathematical Sciences, Professor,  
Department of Information Systems and Computer Modeling,  
Volgograd State University  
yana\_nn@volsu.ru  
<https://orcid.org/0000-0002-6043-9555>  
Prosp. Universitetsky, 100, 400062 Volgograd, Russian Federation

**Mikhail B. Belonenko**

Doctor of Physical and Mathematical Sciences, Professor,  
Department of Forensic Science and Physical Materials Science,  
Volgograd State University  
belonenko@volsu.ru  
<https://orcid.org/0000-0003-2003-3793>  
Prosp. Universitetsky, 100, 400062 Volgograd, Russian Federation

**Abstract.** In this paper, the authors study the influence of a random stress on the parameters of an extremely short optical pulse as it propagates in a nonlinear medium with carbon nanotubes, taking into account the processes of multiphoton absorption and pumping by an external electromagnetic wave. The dependence of the intensity and width of the pulse field on the number of photons, as well as on the amplitude of the pump pulse, is analyzed.

**Key words:** extremely short pulse, carbon nanotubes, multiphoton absorption, random stress.

### Introduction

Extremely short pulses have been the subject of close attention of researchers since the moment they were received for about 40 years. This is due to the unusual nature of their physics and wide practical applications [3; 9], including the study of ultrafast relaxation processes in the microcosm, as well as the interaction of light with matter with a high peak intensity. As is known, the processes of multiphoton absorption (MA) play an important role in the interaction of light with high-intensity radiation. MA is understood as a quantum mechanical process in which several photons are absorbed by an atom or molecule, which gain or lose energy [11]. Interest in MA increased with the advent of lasers, which made it possible to observe this phenomenon in the optical range and advance in the study of nonlinear processes in semiconductor materials [13; 14], as well as in nanostructures [2; 5], including carbon nanotubes (CNTs) [12]. Note that CNTs are attracting more and more attention from the point of view of their practical use in the development of opto-, micro-, and nanoelectronic devices [8; 10]. In this work, we study the dynamics of an extremely short optical pulse in a nonlinear medium with semiconductor carbon nanotubes under random stress, which is important to take into account, since CNTs are randomly stressed even in the general case. For these purposes, we use the model developed earlier, which takes into account multiphoton absorption [7].

### 1. Model and basic equations

We investigate the propagation of a three-dimensional extremely short optical pulse through a dielectric medium containing carbon nanotubes. For carbon nanotubes of the zigzag (m,0) type, the dispersion law can be written as [4]:

$$\epsilon(p, s) = \gamma_0 \sqrt{1 + 4 \cos(ap) \cos\left(\frac{\pi s}{m}\right) + 4 \cos^2\left(\frac{\pi s}{m}\right)}, \quad (1)$$

where  $s = 1, 2, \dots, m$ ,  $\gamma_0 = 2.7$  eV,  $a = 3b/2\hbar$ ,  $b = 0.142$  nm.

Since the field is directed along the CNT axis (for definiteness OZ), then only the component  $A_z(x, y, z, t)$  is nonzero, for the electric current density, respectively  $j_z(x, y, z, t)$ .

Next, we write the three-dimensional wave equation for the nonzero component of the vector potential in a cylindrical coordinate system, and taking into account the gauge:  $\vec{E} = -\frac{1}{c} \frac{\partial \vec{A}}{\partial t}$ :

$$\square A_z + \frac{4\pi}{c} j(A_z) + \Gamma \frac{\partial A_z}{\partial t} - K_p \left(\frac{\partial A_z}{\partial t}\right)^{2n_p-1} = 0, \quad (2)$$

$r, z, \phi$  are coordinates in a cylindrical coordinate system,  $c$  is the speed of light,  $n_p$  is the number of photons,  $K_p$  is the photon absorption coefficient [6],  $\square$  is the d'Alembert operator. The parameter  $\Gamma$  describes the pumping of the electric field:

$$\Gamma = Q_\Gamma \exp\left(-\frac{r^6}{l_\Gamma^6}\right), \quad (3)$$

here  $l_\Gamma$  is the width of the amplifying medium in the direction perpendicular to the direction of propagation of the electric field pulse,  $Q_\Gamma$  is the amplification factor, which depends on the properties of the medium. The expression for the current density along the CNT axis can be written as [2]:

$$j_z = 2e \sum_{s=1}^m \int v(p, s) \cdot F(p, s) dp, \quad (4)$$

where  $e$  is the charge of electron,  $\hbar=1$ , integration is carried out over the first Brillouin zone,  $p$  is the quasi-momentum component of the conduction electron along the CNT axis,  $v(p, s) = \partial\epsilon(p, s)/\partial p$  is the electron velocity,  $F(p, s)$  is the Fermi distribution function. Thus, the effective equation, taking into account the symmetry in the angle ( $\partial/\partial\phi \rightarrow 0$ ) due to the smallness of the accumulated charge due to the inhomogeneity of the field [15]), can be written in the following form:

$$\square A_z + \frac{4en_0\gamma_0 a}{c} \sum_{q=1}^{\infty} b_q \sin\left(\frac{aeqA_z}{c}\right) + \Gamma \frac{\partial A_z}{\partial t} - K_p \left(\frac{\partial A_z}{\partial t}\right)^{2n_p-1} = 0, \quad (5)$$

$n_0$  is the electron concentration,

$$b_q = \sum_s \frac{q}{\gamma_0} a_{sq} \int_{1Bz} \cos(p'q) F(p', s) dp' \int_{-U_0}^{U_0} \frac{\cos(qy)}{\sqrt{2\pi\Delta}} \exp\left(-\frac{(y-U_0)^2}{2\Delta^2}\right) dy, \quad (6)$$

$a_{sq}$  are the coefficients in the expansion of the electron dispersion law (1) in a Fourier series. The last factor here takes into account the random stress with normal distribution.  $U_0$  is the median,  $\Delta$  is the variance of this distribution. In the sum (5), we take into account only the first 10 terms due to the decrease in the coefficients determined by formula (6) with an increase in the number  $q$  [1].

## 2. Numerical simulation and results

The effective equation (5) after reduction to a dimensionless form is solved using numerical methods, taking into account the following initial conditions for the vector potential of the field:

$$\begin{cases} A(z, r, 0) = B \exp\left(-\frac{z^2}{l_z^2}\right) \exp\left(-\frac{r^2}{l_r^2}\right) \\ \frac{dA(r, z, 0)}{dt} = \frac{2v_0 z B}{l_z^2} \exp\left(-\frac{z^2}{l_z^2}\right) \exp\left(-\frac{r^2}{l_r^2}\right) \end{cases}, \quad (7)$$

where  $B$  is the amplitude of the electromagnetic pulse at the initial moment of time,  $l_z, l_r$  is the pulse width along the respective directions,  $v_0$  is the initial impulse velocity along the axis  $z$ . The evolutionary picture of the intensity of an extremely short optical pulse as it

propagates in a dielectric medium with CNTs, taking into account three-photon absorption, is shown in figure 1.

It can be seen from figure 1, that the pulse undergoes broadening, while propagating in a sufficiently localized manner with the division of the main peak into several pulses of different magnitudes. The effect of the number of absorbed photons on the shape and intensity of an extremely short pulse is shown in figure 2.

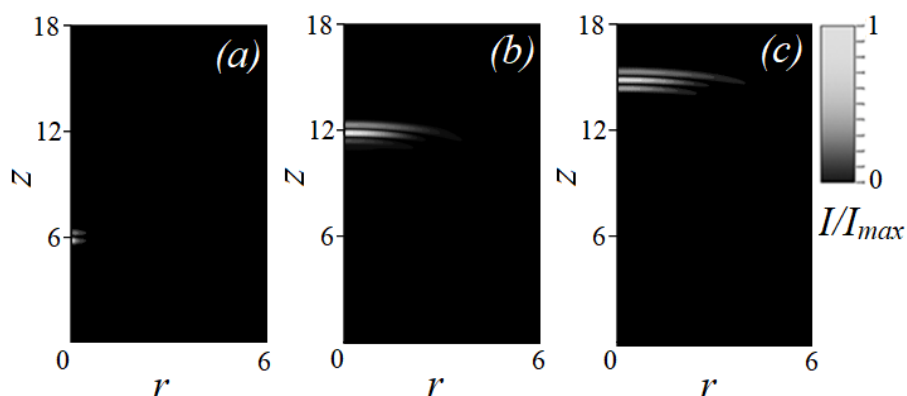


Fig. 1. The dependence of electric field intensity on coordinates ( $Q_{\Gamma} = 0.5$ ):  
 (a)  $t = 0$ ; (b)  $t = 6 \cdot 10^{-14}$  s; (c)  $t = 9 \cdot 10^{-14}$  s.  
 $I_{max}$  is the maximum intensity for each moment of time

According to figure 2, we can conclude that the number of absorbed photons determines not only the shape of the pulse, but also the maximum of its intensity. We also note the appearance of a “tail” behind the main pulse (fig. 2c). Also, based on the two-dimensional picture (fig. 2a, 2b), it can be seen that in the case of two-photon absorption, the pulse experiences not only greater diffraction spreading, but also a curvature of the pulse front due to diffraction. The dependence of the pulse width (the distance at which the intensity decreases by 2 times) on time is shown in the figure 3.

The dependences in figure 3 show that the amplitude of the pump pulse makes it possible to control the transverse width of an electromagnetic pulse propagating in a medium with CNTs. Moreover, the number of absorbed photons also allows us to control this width. Note that over time, in both cases (figures 3a and 3b), the pulse width  $L$  for different numbers of photons becomes the same. We also note that the calculations performed have shown that the random stress parameters do not significantly influence the pulse dynamics. This allows us to speak about the possibility of using the system under consideration in the above conditions. So, for example, in the case of defects in carbon nanotubes (which often occurs in real conditions), stress will appear in CNTs. Thus, such nanotubes can also be used in systems where stable propagation of pulses is required.

## Conclusion

Main conclusions from the study:

- 1) An effective equation that describes the dynamics of an extremely short optical pulse in a medium containing carbon nanotubes, taking into account multiphoton absorption and random stress is derived.

- 2) It is shown that in the model of two-photon and three-photon absorption, taking into account the random stress, the pulses propagate with the preservation of the localization region.
- 3) It was found that the greatest influence on the width of an extremely short optical pulse in the presence of a random stress is exerted by the gain of the pump field.

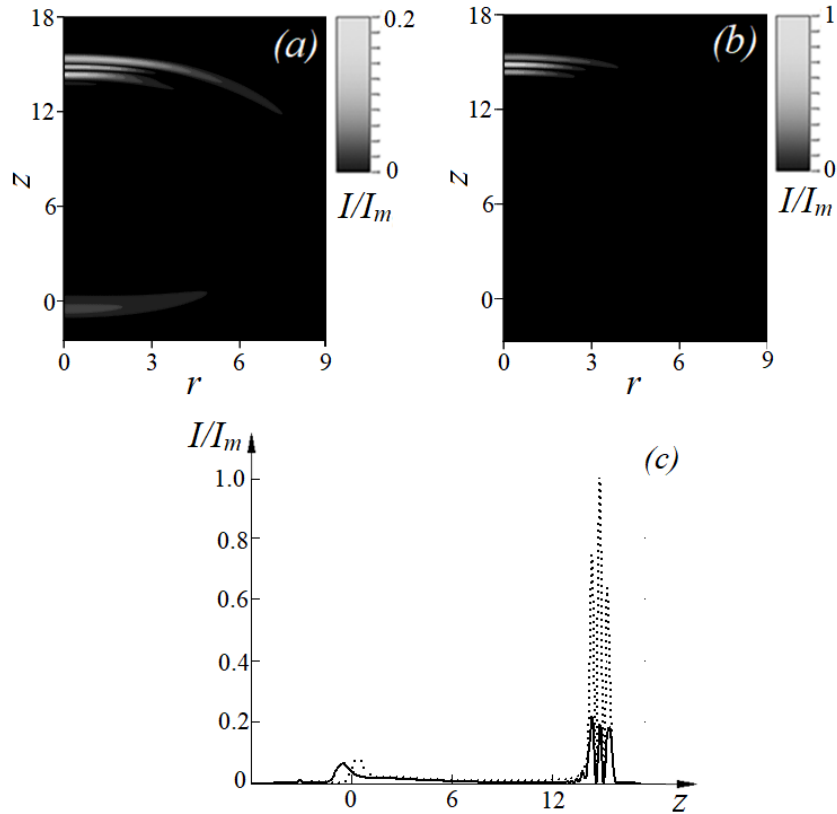


Fig. 2. The longitudinal sections of the intensity from the  $z$  coordinate for different numbers of photons ( $Q_\Gamma = 0, 5$ ,  $t = 9 \cdot 10^{-14}$  s): (a)  $n_p = 2$ ; (b)  $n_p = 3$ ; (c) slices at  $r = 0$ . The solid curve in figure (c) corresponds to case (a), the dotted curve corresponds to case (b). The unit along the  $I$  axis is the intensity  $I_m$  at  $n_p = 3$

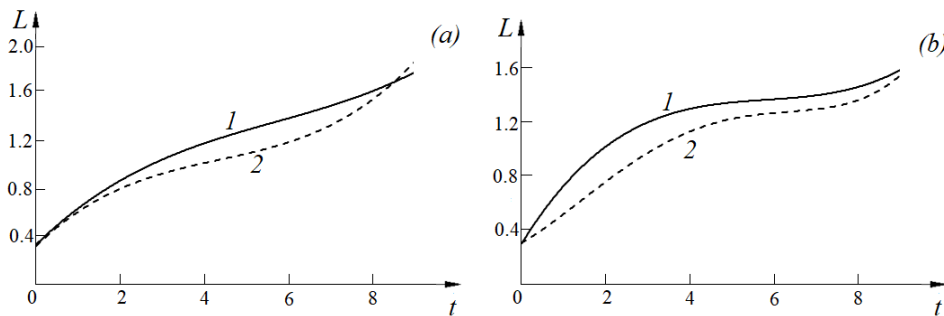


Fig. 3. Time dependence of pulse width for different number of photons  $n_p$ : curve 1 corresponds  $n_p = 2$ ; curve 2 -  $n_p = 3$ . (a)  $Q_\Gamma = 0.1$ ; (b)  $Q_\Gamma = 0.5$

## NOTE

<sup>1</sup> N.N. Konobeeva and M.B. Belonenko express gratitude the Ministry of Science and Higher Education of the Russian Federation for the numerical modeling support under the government task (0633-2020-0003).

## REFERENCES

1. Belonenko M.B., Demushkina E.V., Lebedev N.G. Electromagnetic Solitons in a System of Carbon Nanotubes. *Journal of Russian Laser Research*, 2006, vol. 27, pp. 457-465.
2. Biswas S., Kole A.K., Kumbhakar P. Observation of Two-Photon Induced Three-Photon Absorption in Chemically Synthesized Silver Nanostructures. *Chemical Physics Letters*, 2015, vol. 629, pp. 70-75.
3. Brabec T., Krausz F. Intense Few-Cycle Laser Fields: Frontiers of Nonlinear Optics. *Review of Modern Physics*, 2000, vol. 72, pp. 545-591.
4. Eletsii A.V. Carbon Nanotubes. *Physics-Uspekhi*, 1997, vol. 40, no. 9, pp. 899-924.
5. Goncalves E.S., Cocca L.H.Z., Araujo W.W.R., Parekh K., Oliveira C.L.P., Siqueira J.P., Mendonca C.R., De Boni L., Figueiredo Neto A.M. Influence of Magnetic Field on the Two-Photon Absorption and Hyper-Rayleigh Scattering of Manganese-Zinc Ferrite Nanoparticles. *Journal of Physical Chemistry C*, 2020, vol. 12, pp. 6784-6795.
6. Khalyapin V.A., Bugay A.N. Analytical Approaches to Describing the Dynamics of a Beam Propagating in the Mode of Multiphoton Ionization. *Bulletin of the Russian Academy of Sciences: Physics*, 2022, vol. 86, no. 1, pp. 13-17.
7. Konobeeva N.N., Belibikhin S.V., Belonenko M.B. Extremely Short Pulses in Anisotropic Optical Media with Carbon Nanotubes Taking into Account Multiphoton Absorption. *Optics and Spectroscopy*, 2022, vol. 1301, (in print).
8. Li Y.-T., Sun K., Luo D., Wang Y.-M., Han L., Liu H., Guo X.-L., Yu D.-L., Ren T.-L. A Review on Low-Dimensional Novel Optoelectronic Devices Based on Carbon Nanotubes. *AIP Advances*, 2021, vol. 11, article ID: 110701.
9. Maimistov A.I. Some Models of Propagation of Extremely Short Electromagnetic Pulses in a Nonlinear Medium. *Quantum Electronics*, 2000, vol. 30, no. 4, pp. 287-304.
10. Scarselli M., Castrucci P., De Crescenzi M. Electronic and Optoelectronic Nano-Devices Based on Carbon Nanotubes. *Journal of Physics Condensed Matter*, 2012, vol. 24, no. 31, article ID: 313202.
11. Shen Y.R. The Principles of Nonlinear Optics. *Quantum Electronics*, 2000, vol. 30, no. 4, pp. 287-304.
12. Uryu S., Ajiki H., Ando T. Excitonic Two-Photon Absorption in Semiconducting Carbon Nanotubes within an Effective-Mass Approximation. *Physical Review B*, 2008, vol. 78, no. 11, article ID: 115414.
13. Xing G., Ji W., Zheng Y., Ying J.Y. Two- and Three-Photon Absorption of Semiconductor Quantum Dots in the Vicinity of Half of Lowest Exciton Energy. *Applied Physics Letters*, 2008, vol. 93, article ID: 241114.
14. Yu D., Hu Y.Y., Zhang G., Li W., Jiang Y. Theoretical Studies on the Two-Photon Absorption of II-VI Semiconductor Nanoclusters. *Scientific Reports*, 2022, vol. 12, article ID: 110 (2022). DOI: <https://doi.org/10.1038/s41598-021-04203-w>.
15. Zhukov A.V., Bouffanais R., Fedorov E.G., Belonenko M.B. Three-Dimensional Electromagnetic Breathers in Carbon Nanotubes with the Field Inhomogeneity Along their Axes. *Journal of Applied Physics*, 2013, vol. 114, no. 14, article ID: 143106.

**ПРЕДЕЛЬНО КОРОТКИЕ ОПТИЧЕСКИЕ ИМПУЛЬСЫ  
В УГЛЕРОДНЫХ НАНОТРУБКАХ  
С УЧЕТОМ МНОГОФОТОННОГО ПОГЛОЩЕНИЯ  
В ПРИСУТСТВИИ СЛУЧАЙНОГО НАПРЯЖЕНИЯ**

**Сергей Викторович Белибихин**

Аспирант кафедры судебной экспертизы и физического материаловедения,  
Волгоградский государственный университет  
belibihin@gmail.com  
просп. Университетский, 100, 400062 г. Волгоград, Российская Федерация

**Наталья Николаевна Конобеева**

Доктор физико-математических наук, профессор кафедры информационных систем  
и компьютерного моделирования,  
Волгоградский государственный университет  
yana\_nn@volsu.ru  
<https://orcid.org/0000-0002-6043-9555>  
просп. Университетский, 100, 400062 г. Волгоград, Российская Федерация

**Михаил Борисович Белоненко**

Доктор физико-математических наук, профессор кафедры судебной экспертизы  
и физического материаловедения,  
Волгоградский государственный университет  
belonenko@volsu.ru  
<https://orcid.org/0000-0003-2003-3793>  
просп. Университетский, 100, 400062 г. Волгоград, Российская Федерация

**Аннотация.** В данной работе исследуется влияние случайного напряжения на параметры предельно короткого оптического импульса при его распространении в нелинейной среде, содержащей ахиральные углеродные нанотрубки с полупроводниковой проводимостью. Взаимодействием между электронами соседних углеродных нанотрубок пренебрегаем в силу большого расстояния между трубками. При этом мы учитываем процессы многофотонного поглощения, которые играют важную роль при взаимодействии света с высокоинтенсивным лазерным излучением. Также в работе учитывается и накачка внешней электромагнитной волной, которая является способом компенсации диссипации и обеспечивает стабилизацию предельно короткого оптического импульса. Проанализирована зависимость интенсивности и ширины поля электромагнитного импульса от числа поглощенных фотонов, а также от амплитуды импульса накачки. Проведенные численные расчеты показали, что параметры случайного напряжения не оказывают существенного влияния на динамику импульса. Это позволяет говорить о возможности использования систем с углеродными нанотрубками с возникающим в них напряжением, обусловленным, например, дефектами при разработке оптоэлектронных приборов, основанных на распространении нелинейных электромагнитных волн.

**Ключевые слова:** предельно короткий импульс, углеродные нанотрубки, многофотонное поглощение, случайное напряжение.