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ПЕРСОНАЛИИ

ALEXANDER ASATUROVICH GRIGOR'YAN (devoted to the sixtieth birthday)

Alexander Asaturovich Grigor'yan, a remarkable mathematician, professor at Bielefeld University (Germany), was born on January 14, 1957 in Baku. He became interested in mathematics at school and in 1974 he won the First Award at the 16th International Mathematical Olympiad. This year A.A. Grigor'yan enrolled the Faculty of Mechanics and Mathematics of Moscow State University. He completed his PhD thesis entitled "The properties of harmonic functions on manifolds" under the supervision of Professor Eugene Mikhailovich Landis at Moscow State University in 1982. In 1982 A.A. Grigor'yan joined the Department of Mathematical Analysis and Function Theory at Volgograd State University. Along with research and teaching mathematics, Grigor'yan's mathematical activities included organizing mathematical Olympiads. He ran programs for students who participated in regional, Russian and all-union mathematical Olympiads, where they often won awards. In 1987, he wrote the book 'The assignments of mathematical Olympiads for students' in collaboration with V.A. Sadovnichy and S.V. Konyagin, which is highly recognised in our country and abroad. In 1987 he was awarded a prize of the Moscow Mathematical Society for a series of papers on partial differential equations.

Besides, A.A. Grigor'yan has been awarded the prestigious Whitehead Prize award of the London Mathematical Society for his contribution to the geometric theory of partial differential equations. During his career A.A. Grygor'yan worked in Moscow Aviation Institute (Moscow, Russia), Institute of Management at The Russian Academy of Sciences (Moscow, Russia), Imperial College (London, Great Britain). At present time he is member of faculty at Bielefeld University (Bielefeld, Germany). Since 2007 A.A. Grigor'yan has conducted the well-known Geometric Analysis Seminar in Bielefeld.

Professor A.A. Grigor'yan is widely known for his fundamental contribution to the theory of potential and partial differential equations on Riemannian manifolds, random processes on graphs, theory of functions on fractal spaces. The study of elliptic and parabolic equations on Riemannian manifolds in connection with the geometry of manifold is a relatively new direction in mathematics that is being developed at the intersection of the theory of partial differential equations, function theory, the theory of random processes, and differential geometry. The first works on the subject appeared in the mid-1970s. A.A. Grigor'yan, whose first publications came out in the early 1980's, was a pioneer in this research field in Russia.

The main objective of his research is to investigate the dependency of the global properties of solutions to equations of elliptic and parabolic types on the geometry. The research goals include estimates of the heat kernel, conditions for the fulfillment of Liouville-type theorems, estimates of the eigenvalues of the Laplace and Schrödinger operators, parabolicity conditions for Riemannian manifolds, and so on.

This field has arisen from the problem of classification of noncompact Riemann surfaces. The known problem of identification of the conformal type of a non-compact one-dimensional Riemann surface can be reformulated as follows: does a nontrivial positive superharmonic function exist on this surface? The classification theory of Riemann surfaces and manifolds based on the study of functional spaces (including spaces of harmonic and related functions) on manifolds, was developed in the works of L. Ahlfors, L. Sariot, S.-T. Yau, A. A Grigor'yan and many other mathematicians.

The existence of non-trivial harmonic and superharmonic functions naturally leads to the Liouville-type theorems. A classical formulation of Liouville's theorem asserts that every bounded harmonic function in R^n is the identity constant. The validity of the following statements, also called the Liouville-type theorems, is well known:

- Every positive harmonic function in v is identical with constant (p-Liouville property).
- Every function that is harmonic in R^n and has a finite Dirichlet integral is identical with constant (D-Liouville property).
- If $u \in L^p(R^n)$ is a harmonic function and $1 \leq p < \infty$, then $u \equiv 0$.

Traditionally, the following approach to theorems of the Liouville type is applied. Suppose that a class of functions A and an elliptic operator L are given on the Riemannian manifold L. We say that the property (A, L) is satisfied on M if any solution of the equation $Lu = 0$ belonging to the functional class A is the identity constant. We note that recently there has been a trend towards a more general approach to Liouville-type theorems, namely, the dimensions of various spaces of solutions of elliptic equations are estimated.

A.A. Grigor'yan started the study of asymptotic properties of harmonic functions in relation to the geometry when he was a student. In his first, unfortunately not very well-known paper [1], he studied the connections between the geometry of the domains of revolution in R^n and the rate at which infinity harmonic functions tend to infinity using the inequality of Harnack. This subject related to the conditions for the fulfillment of the Harnack inequality for the Laplace — Beltram equation and the heat equation on Riemannian manifolds and, as a consequence, the fulfillment of the Liouville-type theorems, was substantially developed both in early and later papers of A.A. Grigor'yan (see [2; 3; 4]).

A.A. Grigor'yan has more than 100 papers published in leading mathematical journals. Many of the research A.A. Grigor'yan conducted are devoted to the criteria and geometric conditions of parabolicity of Riemannian manifold type. One of the most significant and well-known results in this direction is the criterion of parabolic type [5], which asserts that a Riemannian manifold has a parabolic type if and only if the capacity of some (any) compactum in it is equal to zero. The search for signs of parabolic type has a long history. A good overview of contemporary research in this subject is presented in the well-known review [6].

A.A. Grigor'yan studied the behavior of harmonic functions with a finite Dirichlet integral. In particular, the criterion for the existence of a bounded harmonic function with a finite Dirichlet integral on a Riemannian manifold was proved, and also a method for finding the dimension of the spaces of such functions was found [7]. A significant number of works A.A. Grigor'yan devoted to finding of the geometric conditions for the validity of Liouville-type theorems for solutions of the stationary Schrödinger equation and, as a consequence, the geometric conditions for the stochastic completeness of Riemannian manifolds [8; 9].

Another fundamental and significant area of research presented in the works of A.A. Grigor'yan is estimation of the fundamental solutions to the heat equation for a wide class of manifolds, including graphs and metric spaces [29; 45; 41; 52; 68]. In addition, a number of studies have been devoted to problems of random walks on graphs, diffusion processes on fractal sets, and so on [28; 48; 63].

A.A. Grigor'yan has been an invited speaker at numerous World and European Mathematical Congresses.

An internationally recognised scholar Alexander Asaturovich Grigor'yan has a wonderful personality and his professional expertise is matched by consideration, responsiveness, ever-present warmness and boundless kindness. The Editorial Board wishes him wholeheartedly health and further achievements.

Here is the list of the most remarkable papers of A.A. Crigor'yan:

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